

*First days of a logic course*¹

Los primeros días de un curso de lógica

John Corcoran
corcoran@buffalo.edu
UNIVERSITY AT BUFFALO
Buffalo, NY, USA

Fecha de recepción: 10-03-17
Fecha de aceptación: 13-06-17

Abstract: This short paper sketches one logician's opinion of some basic ideas that should be presented on the first days of any logic course. It treats the nature and goals of logic. It discusses what a student can hope to achieve through study of logic. And it warns of problems and obstacles a student will have to overcome or learn to live with. It introduces several key terms that a student will encounter in logic.

A *proposition* is either true or false *per se*, not "for this or that person". An *argument* is either valid or invalid *per se*, not "for this or that person". An *argumentation* is either conclusive or inconclusive, not *per se*, but *for a person*.

However, that a given argumentation is conclusive for a given person is undeniably a matter of that person's subjective thoughts—but only in certain respects: if a given argumentation is conclusive for one person but not for another, the first knows something the second doesn't know. And not every argumentation thought by a given person to be conclusive for that person actually is conclusive for that person. There is more to an argumentation's conclusiveness than subjectivity. Suggested readings are given in parenthetical citations keyed to the References list.

Keywords: Logic, Argumentation, Begging-the-question, Demonstration, Deduction.

Resumen: En este corto artículo se bosqueja la opinión de un lógico sobre ciertas ideas básicas que deberían explicarse en los primeros días de cualquier curso de lógica. Se aborda la naturaleza y los objetivos de la lógica. Se discute lo que un estudiante puede esperar lograr estudiando lógica y se

¹ **NOTA DEL DIRECTOR DE QUADRIPARTITA RATIO:** Como el autor lo menciona en la sección de agradecimientos (ver al final del artículo), la traducción al español de una primera versión de este artículo fue publicada en 2010 en la revista *Ergo* de la Facultad de Filosofía de la Universidad Veracruzana (*Ergo. Nueva Época*, 25, 31-45. ISSN: 0187-6309), bajo la dirección de Ariel Campirán. Esta nueva versión, que aparece en su idioma original, incluye adiciones y precisiones importantes, tanto en cantidad como en contenido, y es por tal motivo que nos pareció pertinente ofrecerla a los lectores de *Quadripartita Ratio*. Siendo Corcoran un investigador de primer orden en el área de la lógica, pero también un docente crítico y comprometido, consideramos fundamental poner este artículo a la disposición de las personas interesadas para que puedan constatar las diferencias entre las dos versiones y evaluar por sí mismas su importancia. Y aprovechamos, desde luego, esta nota para agradecer a nuestros amigos de *Ergo* por la amable autorización para la publicación del material ya aparecido en el número mencionado.

advierte sobre los problemas y obstáculos que tendrá que vencer o con los cuales tendrá que aprender a vivir. Se introduce una buena cantidad de términos clave que un estudiante encontrará en lógica.

Una *proposición* es o verdadera o falsa *per se*, no “para esta o aquella persona”. Un *argumento* es o válido o inválido *per se*, no “para esta o aquella persona”. Una *argumentación* es o concluyente o no concluyente, no *per se*, sino *para una persona*.

Sin embargo, el hecho de que una argumentación dada sea concluyente para una persona determinada depende sin lugar a dudas de las ideas subjetivas de tal persona, pero sólo hasta cierto punto: si una argumentación dada es concluyente para alguien pero no para otro, el primero sabe algo que el segundo no sabe; y no toda argumentación que alguien en específico considere concluyente para tal persona es en realidad concluyente para tal persona. En el carácter concluyente de una argumentación hay más cosas involucradas que sólo la subjetividad.

Se proporcionan lecturas sugeridas en las referencias parentéticas, ligadas con la bibliografía final.

Palabras clave: Lógica, Argumentación, Petición de principio, Demostración, Deducción.

Introduction

On the first day of a logic course, the teacher should remind the students that most *words* are *ambiguous* (have more than one *meaning*): the two *five-letter* words ‘logic’ and ‘proof’ are used in multiple *senses* even in a logic course. The senses recommended here are not the only useful choices: students should know this to be able to benefit from other logicians. And even in logic attention to the figurative/literal distinction pays handsome dividends. Moreover, italics, single quotes, and double quotes are used to mark different important distinctions. Students should be encouraged to ask about terminology so that it will serve them and not intimidate them (Corcoran, 1999b; 2009; and 2016).

The teacher should also emphasize that the subject many of us call *logic* was developed by human beings and that it is still being developed. Logic is not settled, not even to the extent that number theory is. Competent, established modern logicians disagree on fundamental points although there are significant areas of wide agreement. Over the years new generations of logicians renovated and expanded the work of their predecessors. Many—but not all—*propositions* believed to be *true* by earlier

logicians are *disbelieved* by many modern logicians. Besides, historians of logic are continually reinterpreting the records left by earlier logicians (Corcoran, 2010a).

Logic was developed by human beings in response to human needs. Especially important among such needs is the need to be able to distinguish (genuine) *proofs* from bogus “proofs”: to tell conclusive *argumentations* from fallacious *argumentations*. Proofs are also known as demonstrations. Every proof has a true *conclusion*; many a “proof” has a false conclusion. There is no way to prove a false proposition. But most importantly, the teacher should patiently explain in general terms what a proof is, how a proof is made, and how an attempt to make a proof can go wrong. Logic is primarily about proofs. More exactly, the nature of proofs is one of the things logic is about. Without focus on proofs, logic would be empty.

A logic course should respond to student needs. Learning logic requires discipline, patience, objectivity, and focus. But the humanistic and spiritual nature of logic should not be left out (Corcoran, 1989b; Corcoran & Frank, 2013).

In founding logic, Aristotle built on the Socratic distinction between *believing* a proposition to be true and *knowing* that it is true: the belief-knowledge distinction. Proof produces *knowledge*, not just belief, not *opinion*. Persuasion produces *opinion*, belief that is not knowledge. My knowledge consists of my beliefs that I *know* to be true. My opinions are my beliefs that I *do not know* to be true (Corcoran, 2006a; Corcoran & Hamid, 2015).

Every *proposition* is either true or false *per se*. Not every *proposition* is either known to be true or known to be false by a given person. The famous Goldbach Hypothesis is one of many *counterexamples* to the proposition that every proposition is either known to be true by someone or known to be false by someone (Corcoran, 2005, 2017).

An *argumentation* is a three-part system composed of a “premise set”, a “conclusion”, and a step-by-step “chain-of-reasoning”. In order for an argumentation to be conclusive, to be a proof (of its conclusion to a given group of people), it is *necessary and sufficient* for the premises to be known to be true (by the group, by every member) and for its chain-of-reasoning to show (to the group) that its *conclusion* is a *logical consequence* of its *premises*. To be a proof (of its conclusion to a given group of people) it is *necessary but not sufficient* for an argumentation to have all true premises and to have a true conclusion *implied by* the premises. In order for an argumentation to be *inconclusive* (*fallacious* or be a bogus “proof”) (to a given group of people) it is necessary and sufficient either for its premises to be not all known to be true (by the group) or for its chain-of-reasoning to not show (to the group) that its conclusion is a logical consequence of its premises. Every fallacious “proof” has faulty premises or faulty reasoning. To be fallacious in this sense, it is *sufficient but not necessary* for an argumentation to have a false premise or a conclusion not implied by the premises (Corcoran, 1989a).

An argumentation is *cogent* (to a given group of people) if its chain-of-reasoning shows (to the group) that its conclusion is a logical consequence

of its premises, non-cogent otherwise. To be a proof (of its conclusion to a given group of people) it is *necessary but not sufficient* for an argumentation to be cogent (to the group). *Critical evaluation of an argumentation to determine whether it is a proof for a given person reduces to two basic issues: are the premises known to be true by the given person? And does the chain of reasoning deduce the conclusion from the premise-set for the given person?*

This is not a relativist or subjectivist view of proof: if a given argumentation is a proof to one person but fallacious to another, then the first has objective knowledge that the second lacks (Corcoran & Hamid, 2014).

What is logic about?

As said, logic is primarily about proof. In its quest to understand proof, logic involves everything that must be understood to understand proof. Again as said, logic presupposes the Socratic distinction between believing a proposition to be true and knowing that it is true: the belief-knowledge distinction. Not every proposition believed to be true is really true. But every proposition known to be true is really true. Every proposition proved to be true is known to be true.

Logic is about proof—not persuasion, and not faith. Persuasion is studied in *rhetoric*—a field that needs logic as much as mathematics needs logic. Before a proof can be started, we must have in mind a “conclusion” to be proved and we must have in mind the “premise set” that the proof will be based on. The conclusion is often a proposition believed or conjectured but not known to be true. Proof often transforms opinion into knowledge. The propositions in the premise set must be known to be true by anyone who will be able to *possess* the argumentation as a proof.

In the context of a first course in logic, it would be natural to say that logic is proof theory, the theory of proof, just as it is natural to call arithmetic number theory, the theory of numbers. But the two-word expression ‘proof theory’ has been given a much narrower meaning in advanced logic

where it denotes, not a study of proofs, but a study of character strings some of which can be taken to be descriptions of proofs (Corcoran, 1973: 28f; and Corcoran, Frank & Maloney, 1974: 625ff).

Every so often we need to remind students that we are not infallible, but that doesn't mean we don't have knowledge. Moreover, students need reminding that we sometimes get carried away a tad and that we sometimes need to fudge (Corcoran, 1999b).

Begging-the-question

One obvious fact about any actual proof is that its premises are known to be true by those who possess it: those for whom it is a proof. The premises are “established fact”.

The fallacy of accepting as a proof an argumentation whose premises are not known to be true is traditionally called *begging-the-question*. It is not to the point to say how the tradition got started but it does help students to note that this bizarre expression does not use either the word ‘begging’ or the word ‘question’ in their most familiar senses. I hyphenate it to encourage students to understand it as a unitary expression whose meaning is not derived from the meanings of its parts.

Begging-the-question includes but is not exhausted by “assuming what is to be proved”, as long as ‘assuming’ means “explicitly or implicitly taking as a premise for purposes of proving” (Corcoran & Frank, 2015). There is something perverse about using the etymologically obscure expression ‘begging-the-question’ for “assuming what is to be proved” in the above sense if only because the latter fallacy already has a clear name: *circular reasoning*. Besides there is nothing wrong about assuming what is to be proved *per se*: we often try to understand a proposition better by assuming it in the process of deducing its consequences. In fact, this is a key step in the *method of analysis*: a method for discovering proofs (Corcoran, 1979).

In order to serve as premises of a person's proof, propositions must be known to be true by that person: a proof can use propositions now known to

be true to gain knowledge of a conclusion not now known to be true (Corcoran, 1989a).

This fallacy has other names as well but it is far more often called ‘begging-the-question’ than called anything else. Many people who do not understand proof use the ambiguous expression ‘begging-the-question’ *only* in other senses. Those who insist on using ‘begging-the-question’ in any other way should be prepared to say what they call *this* fallacy, which is one of the most important fallacies—if it is not the most important fallacy. It is surely the most general *material* fallacy. It deserves a familiar name. Alternatives to ‘begging-the-question’ include the following: ‘faulty assumption (or premise)’, ‘unwarranted assumption’, ‘uncertain assumption’, and ‘unsecured basis’—none of which have the connotation or impact carried by ‘begging-the-question’.

To avoid this fallacy, check your premises. Taking possession of a proof requires effort.

The English language conveniently enables us to distinguish between *having proof of* (including sufficient evidence for) a proposition and *having a proof* (demonstration) of a proposition. We have proof of any proposition we know to be true regardless of whether we have a proof of it. Knowers necessarily have proof of all of the premises of their proofs, even if all of those premises are self-evident truths lacking proofs. A truth is self-evident *to a knower* if that knower does not need a proof, *i.e.*, if that knower knows it without a proof—by “looking at the fact”. Such knowledge is non-demonstrative. In ideal cases, a person who knows a self-evident truth has proof but not a proof. Of course, it can happen that a person who knows a truth without a proof may later find a proof. And conversely, it can happen that a person who knows a truth by means of a proof may later find non-demonstrative proof. A deduction having a premise lacking proof is not a demonstration: as said above, taking such a deduction as a proof is committing the fallacy known by logicians as *begging-the-question* (Corcoran, 1989a). Discussion of *begging-the-question* and self-evident truths provides another opportunity to remind students

about ambiguity. Expressions similar to technical terms in logic are used outside of logic with different meanings: In logic ‘beg-the-question’ doesn’t mean ‘raises the question’ or ‘evades the question’; ‘self-evident’ doesn’t mean ‘obvious’, ‘unquestionable’, or ‘trivial’.

Knowledge Neutrality of Deduction

Another point to be made about proofs is that the same process of *deduction* used to *infer* the conclusion from established truths is also used to *deduce* conclusions from premises not known to be true or even from premises known to be false. Deduction produces knowledge of implication of a conclusion by premises. Inferences, *i.e.*, proofs, are deductions from premises known to be true. Inferences, in addition, produce knowledge of a conclusion’s truth (Corcoran, 2006b).

In a slogan: demonstration produces knowledge of truth; deduction produces knowledge of implication.

An *argument* (more fully, a *premise-conclusion argument*) is a two-part system composed of a set of propositions and a single proposition: its “premises” and its “conclusion”. Every argument is completely determined by its premises and its conclusion. Here are three arguments in the same logical form.

<i>Argument 1</i>	<i>Argument 2</i>	<i>Argument 3</i>
Every square is a rectangle.	Every quadrangle is a square.	Every triangle is a rectangle.
<u>No rectangle is a circle.</u>	<u>No square is a rhombus.</u>	<u>No rectangle is a square.</u>
No square is a circle.	No quadrangle is a rhombus.	No triangle is a square.

One way of *inferring* the conclusion of argument 1 from its premises involves “looking at an arbitrary square”. But there are many other chains-of-reasoning from those premises to that conclusion. Every one of the processes of deduction that could have been used to *infer* the true conclusion of argument 1 from its known true premises could also have been used to *deduce* the false conclusion of argument 2 from its premises, one of which is false. Likewise the same processes of deduction could have been used to deduce the true conclusion of argument 3 from its false premises.

Some people try to make this point by saying that logicians do not care whether the premises are true. Apart from the absurdity of what is said about logicians, the point is not even about logicians: it is about implication. Here ‘implication’ refers to the relation of premises to any conclusion implicit in them, that is, to any conclusion that conveys no information not conveyed by the premises. Deduction produces knowledge of implication; deduction is information processing, the process of coming to know that the information conveyed by the conclusion is already conveyed by the premises (Corcoran, 1996).

Direct deductions

As we saw, every argument is completely determined by its premises and its conclusion. In the sense often used in logic but rarely used elsewhere, recall that an *argument* is a two-part system composed of a set of propositions and a single proposition: its premises and its conclusion. Again in the sense often used in logic but rarely used elsewhere, an argument is *valid* if the conclusion follows from its premises, *invalid* otherwise. In contrast, over and above the premises and conclusion, every proof has a chain-of-reasoning that shows that the (*final*) *conclusion* follows

Content-Correspondence Table

Argument	Term 1	Term 2	Term 3	Premises	Conclusion
1	square	rectangle	circle	TT	T
2	quadrangle	square	rhombus	FT	F
3	triangle	rectangle	square	FF	T

logically from the premises. There are many different proofs with the same conclusion and the same premises but different chains-of-reasoning.

There are many kinds of proof. But it is instructive to look at the two simple types of proof that Aristotle studied: the direct and the indirect (Corcoran, 2009b).

A direct proof based on three premises p1, p2, and p3 and having a chain-of-reasoning with three intermediate conclusions ic1, ic2, and ic3 can be pictured as below. The final conclusion fc occurs twice: once as a goal to be reached and then as an accomplished goal. Some intermediate conclusions can equally well be called intermediate premises. Since every proof is a deduction of its conclusion from its premises, the same picture illustrates a deduction.

Three-premise Four-step Direct Deduction Schema

p1
 p2
 p3
 ?fc
 ic1
 ic2
 ic3
 fc
 QED

Direct deduction 1

1. Every square is a rectangle.
2. Every rectangle is a polygon.
3. No circle is a polygon.
- ? No square is a circle.
4. No polygon is a circle. 3
5. Every rectangle is a polygon. 2
6. No rectangle is a circle. 5, 4
7. Every square is a rectangle. 1
8. No square is a circle. 7, 6

QED

Indirect deductions

An indirect proof based on three premises p1, p2, and p3 and having a chain-of-reasoning with three intermediate conclusions ic1, ic2, and ic3 can be pictured as below. After the final conclusion has

been set forth as a goal, an exact contradictory opposite *fc called the *reductio assumption* is added as a new assumption. Taking @ to mean “assume (for purposes of reasoning)”, the first line after the goal is @*fc, where a contradictory opposite of the conclusion is assumed as an auxiliary “premise”.

From the premises augmented by the *reductio* assumption, intermediate conclusions are deduced until the reasoner notes that the last intermediate conclusion, ic3 in this example, contradicts one of the previous “lines”—often one of the premises, sometimes a previous intermediate conclusion, sometimes even the *reductio* assumption, and, in very rare cases, itself, when the last intermediate conclusion is a self-contradiction such as “one is not one”. The fact that the reader notes the contradiction is often expressed by writing the words ‘This is a contradiction’, ‘A contradiction’ or even just ‘Contradiction’. Here in our diagram we use X, the capital ecks.

Three-premise Four-step Indirect Deduction Schema

p1
 p2
 p3
 ?fc
 @*fc
 ic1
 ic2
 ic3
 X
 QED

Indirect Deduction 1

1. Every square is a rectangle.
2. Every rectangle is a polygon.
3. Some circle is not a polygon.
- ? Some circle is not a square.
4. Assume every circle is a square.
5. Every circle is a rectangle. 4, 1
6. Every circle is a polygon. 5, 2
7. But, some circle is not a polygon. 3
8. Contradiction. 7, 6

QED

The fallacy of accepting as a proof an argumentation whose chain-of-reasoning does not establish that the conclusion follows from the premises could be called ‘begging-the-deduction’. But it is called *faulty reasoning*, *fallacious chain-of-reasoning*, *fallacious derivation*, or *non-cogent derivation*. This is the most general *formal* fallacy. To avoid the fallacy of faulty reasoning: check your individual steps of reasoning, make sure you used only your stated premises, and make sure your chain-of-reasoning reached your proposed conclusion—and not some other proposition.

Not every argumentation thought to be a proof actually is a proof

As said above, in order for an argumentation to be a proof (of its conclusion to a given group of people) it is necessary and sufficient for the premises to be known to be true (by the group) and for its chain-of-reasoning to show (to the group) that its conclusion is a logical consequence of its premises.

Likewise, in order for an argumentation to be a fallacious (to a given group of people) it is necessary and sufficient for the premises to be not all known to be true (by the group) or for its chain-of-reasoning to not show (to the group) that its conclusion is a logical consequence of its premises.

Every fallacious “proof” has faulty premises or faulty reasoning. Every fallacious “proof” begs-the-question or begs-the-deduction. Some argumentations are fallacious for everyone: those with false premises, those whose conclusions do not follow from their respective premises sets, and those with other disqualifications.

If the chain-of-reasoning shows that the conclusion follows but the premises are not all known to be true, then the argumentation is a *deduction* but not a proof. Aristotle said many years ago: “every proof is a deduction but not every deduction is a proof”. If the premises of a deduction come to be known to be true, the deduction comes to be a proof. The order does not matter. You can construct a proof, or demonstration, by securing knowledge of the premises and then establish that the conclusion follows from them. You can

construct a proof by establishing that the conclusion follows from the premises and then securing knowledge of them.

In typical cases, some or all of a demonstration’s premises have been demonstrated. The demonstrations of the premises may be thought of as added to the demonstration making *its extended demonstration*. This process may be continued until we arrive at *its fully extended demonstration* whose premises are propositions known to be true by the demonstrator without using deduction. These ultimate premises of the fully extended deduction may be called *its axioms*—regardless of whether they happen to have been adopted previously as axioms in some axiomatized theory. A given branch of mathematics may be axiomatized in various ways, an axiom of one axiomatization being a deduced theorem of another (Corcoran, 1999a).

Hidden consequence and hidden independence

Two related problem types are central to logic: consequence problems—discussed above, but not by that name—and independence problems. *Consequence problems* have the form: to show that a given conclusion is a consequence of a given premise set—if it is. *Independence problems* have the form: to show that a given conclusion is *not* a consequence of a given premise set—if it is not. Traditionally, a proposition, not a consequence of a set of propositions, is said to be *independent* of the latter, a terminology whose awkwardness needs to be pointed out to students (Corcoran, 2015, 2010b).

A lengthy *deduction* that Andrew Wiles discovered shows the Fermat conjecture to be a consequence of arithmetic axioms. Consequence problems were solved by *deduction*: deducing the conclusion from the premises using a series of deductively evident steps. The branch of mathematical logic that is most relevant to consequence problems is called proof theory (Corcoran, 1973).

Reinterpreting ‘number’, ‘zero’, and ‘successor’ so as to produce true propositions from the other two axioms and a false proposition from the Mathematical Induction axiom shows the latter to be

independent of the other two axioms of Gödel's 1931 axiomatization of arithmetic. Independence problems were solved by *reinterpretation*: reinterpreting non-logical constants so as to produce true premises and false conclusion. There is not time in the first days to treat independence problems, but they will be dealt with extensively on later days. The branch of mathematical logic that is most relevant to independence problems is called model theory (Corcoran, 1973).

A proposition that is a consequence of (or is independent of) a premise set is said to be a *hidden consequence* (or a *hidden independence*) if it is not obviously such. Without hidden consequence, deduction would be pointless. Without hidden independence, independence proof would be pointless. Hidden consequence and hidden independence are basic for justifying the study of logic and, indeed, for justifying the existence of logic as a field. This points to another human need that logic was developed to satisfy (Corcoran, 2010b).

Above we noted that logic was developed by human beings in response to human needs. One need that initiated logic was probably the need to be able to distinguish (genuine) *proofs* from bogus "proofs": to distinguish cogent *argumentations* from fallacious *argumentations*. But, the pursuit of this goal reveals the need to determine of a given conclusion whether it follows from given premises. Without hidden consequence and hidden independence no such need would ever arise.

Appendix

Applying Logical Terminology in History

There are many propositions about right triangles. Your students probably know that (1) the square on one side of a right triangle is equal to the sum of the squares on the other two sides. This follows from the Pythagorean Theorem, as is easily seen. It is also a fact that (2) the square on one side of a right triangle is equal to the difference between the squares on the other two sides. This is related to the fact that, given any three quantities, if the first is the sum of others, then the second is equal to the difference between the first and the third and, of course, also

the third is equal to the difference between the first and the second. It is also a fact that (3) the square on any one side of a right triangle not equal to the sum of the squares on the other two sides is equal to the difference between those two squares. It is also a fact that (4) the square on any one side of a right triangle not equal to the difference between the squares on the other two sides is equal to sum of those two squares.

If Pythagoras proved any one of these four propositions, he did so by deducing it from propositions he knew to be true. If he deduced it from propositions he did not know to be true, he did not have a proof: he had a question-begging deduction—he begged-the-question.

It is not necessary to know that the last two of the four propositions, (3) and (4) are both true in order to know that each implies the other. If Pythagoras knew that one implies the other, he deduced one from the other.

N. Bourbaki, the legendary mathematician, was not far from the above view when he wrote the following (Corcoran, 1973: 23):

By a proof, I understand a section of a mathematical text [...]. Proofs, however, had to exist before the structure of a proof could be logically analyzed; and this analysis [...] must have rested [...] on a large body of mathematical writings. In other words, logic, so far as we mathematicians are concerned, is no more and no less than the grammar of the language which we use, a language which had to exist before the grammar could be constructed [...]. The primary task of the logician is thus the analysis of the body of mathematical texts.... (Bourbaki, 1949).

Acknowledgements

This paper is a revised and expanded version of a previous paper that was translated into Spanish.

Corcoran, John. (2010). Los primeros días de todo curso de Lógica. *Ergo. Revista de Filosofía de la Universidad Veracruzana*. 25, 31–45. Spanish translation by Patricia Diaz-Herrera.

The previous paper originated as a handout for the first day of the fall 2007 version of my graduate course *Introduction to Logic for Advanced Students*. Former students in that course, especially Paul Penner and Patricia Diaz-Herrera, deserve much credit for their contributions. Penner spent long hours ferreting out errors, omissions, and places that needed work.

The existence of the paper is due almost exclusively to Diaz-Herrera's initiative and dedication. Making the paper from the handout was her idea. And she undertook the translation. I am very grateful to her.

For encouraging responses, useful suggestions, and critical insights, my warm thanks to

the following: J. Baldwin, G. Boger, G. Burda, Jos. Corcoran, M. Cowen, P. Crivelli, T. Drucker, C. Ende, L. Estrada, I. Hamid, F. Hansen, J. Henderson, L. Jacuzzo, J. Keller, J. Kreiss, J. Legault, R. Lewin, Y. Liu, R. Main, D. Marans, C. Martínez-Vidal, H. Masoud, K. Miettinen, J. Miller, M. Mulhern, F. Nabrasa, P. Penner, S. Pererê, D. Plache, M. Rind, G. Rising, A. Schremmer, T. Sullivan, C. Ullman, R. Weaver, L. Woomer, S. Ziewacz, and others.

M. Cerezo, L. Estrada, M. Mulhern, R. Lewin, J. M. Sagüillo, A. Zela, and others were consulted on the translation.

References

- BOURBAKI, N. (1949). Foundations of Mathematics for the Working Mathematician. *Journal of Symbolic Logic*, 14(1), 1-8.
- CORCORAN, J. (1973). "Gaps between logical theory and mathematical practice". In M. BUNGE (ed.). *Methodological Unity of Science* (pp. 23–50). Dordrecht: Reidel.
- CORCORAN, J. (1979). Review of "Hintikka, J. and U. Remes. *The Method of Analysis*". *Mathematical Reviews*, 58, 3202.
- CORCORAN, J. (1989a). Argumentations and logic. *Argumentation*, 3, 17–43.
- CORCORAN, J. (1989b). Inseparability of Logic and Ethics. *Free Inquiry*, Spring, 37–40.
- CORCORAN, J. (1995). Semantic Arithmetic: a Preface. *Agora*, 14, 149–156.
- CORCORAN, J. (1996). "Information-theoretic logic". In C. MARTÍNEZ *et al.* (eds.). *Truth in Perspective* (pp. 143–176), Aldershot, UK: Ashgate,
- CORCORAN, J. (1999a). Axiomatic method. *Cambridge Dictionary of Philosophy*. Cambridge: Cambridge University Press.
- CORCORAN, J. (1999b). Critical thinking and pedagogical license. *Manuscrito*, 22, 109–16.
- CORCORAN, J. (2005). Counterexamples and proexamples. *Bulletin of Symbolic Logic*, 11, 460.
- CORCORAN, J. (2006a). An Essay on Knowledge and Belief. *The International Journal of Decision Ethics*, 2(2), 125-144.
- CORCORAN, J. (2006b). C. I. LEWIS: History and Philosophy of Logic. *Transactions of the C. S. Peirce Society*. 42, 1–9.
- CORCORAN, J. (2007a). "A priori/a posteriori". In J. LACHS & R. TALISSE (eds.). *Encyclopedia of American Philosophy*. New York: Routledge.
- CORCORAN, J. (2007b). "Knowledge and Belief". In J. LACHS & R. TALISSE (eds.). *Encyclopedia of American Philosophy*. New York: Routledge.
- CORCORAN, J. (2009a). Ambiguity: lexical and structural. *Bulletin of Symbolic Logic*, 15, 235–6.
- CORCORAN, J. (2009b). Aristotle's Demonstrative Logic. *History and Philosophy of Logic*, 30, 1–20.
- CORCORAN, J. (2009c). "Sentence, proposition, judgment, statement, and fact". In W. CARNIELLI *et al.* (eds.) *The Many Sides of Logic* (pp. 71–103). London, College Publications.
- CORCORAN, J. (2010a). Essay-Review of "Striker, G., trans. 2009. *Aristotle's Prior Analytics: Book I*. Trans. with Intro. and Comm. Oxford: Oxford University Press. 2005". *Notre Dame Philosophical Reviews*.
- CORCORAN, J. (2010b). Hidden consequence and hidden independence. *Bulletin of Symbolic Logic*, 16, 443.
- CORCORAN, J. (2010c). Los primeros días de todo curso de Lógica. *Ergo. Revista de Filosofía de la Universidad Veracruzana*, 25, 31–45. (Spanish translation by Patricia Diaz-Herrera of a previous version of this paper).
- CORCORAN, J. (2015). Teaching independence. *Bulletin of Symbolic Logic*, 21, 101–102.
- CORCORAN, J. (2016). Logic teaching in the 21st century. *Quadripartita Ratio: Revista de Argumentación y Retórica*, 1(1), 1–34.
- CORCORAN, J. (2017). Review of "Paseau, Alexander, 'Knowledge of mathematics without proof', *British J. Philos. Sci.* 66 (2015), no. 4, 775–799". *Mathematical Reviews*.
- CORCORAN, J., & W. FRANK (2013). Cosmic Justice Hypotheses. *Bulletin of Symbolic Logic*, 19, 253.
- CORCORAN, J., & W. FRANK. (2015). Assumptions: illative and typographical. *Bulletin of Symbolic Logic*, 21, 364–5.
- CORCORAN, J., W. FRANK & M. MALONEY (1974). String theory. *Journal of Symbolic Logic*, 39, 625–637.
- CORCORAN, J., & I. S. HAMID (2014). Objectivity-subjectivity distinctions. *Bulletin of Symbolic Logic*, 20, 248.
- CORCORAN, J., & I. S. HAMID (2015). Investigating knowledge and opinion. In A. BUCHSBAUM & A. KOSLOW (eds.). *The Road to Universal Logic. Vol. I* (pp. 95-126). Basel: Springer.